On the feasibility of transmission scheduling in a code-based transparent passive optical network architecture

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On the long-term, the ever-increasing demands for bandwidth can only be satisfied by the introduction of optical network technologies in the local loop. Here, a code-based transparent passive optical network is considered in which nodes may asynchronously transmit data to each other with a risk on collisions. In this work, the feasibility of transmission scheduling is evaluated in order to resolve contention at the passive couplers. The influence of several network dimensioning parameters is evaluated such as fiber length, traffic load, maximum number of active users, and the operational bit rate in the network. These are studied for different burst size distributions namely Geometric and heavy-tailed distributions. The study shows that the feasibility of the transmission scheduling depends on these parameters. Indeed, they directly impact the topology design and the architecture of the network. The results of this work may apply to all code-based systems that have full-bit duration codes and that encode each bit.

1. Introduction

1.1 Architecture and optical codes

The architecture considered in this paper has been introduced in [1] as a possible implementation of an optical access tier. As shown in Figure 1, multiple passive optical networks (PONs) are connected to a single central office (CO) and each PON contains N_i optical networking units (ONUs).

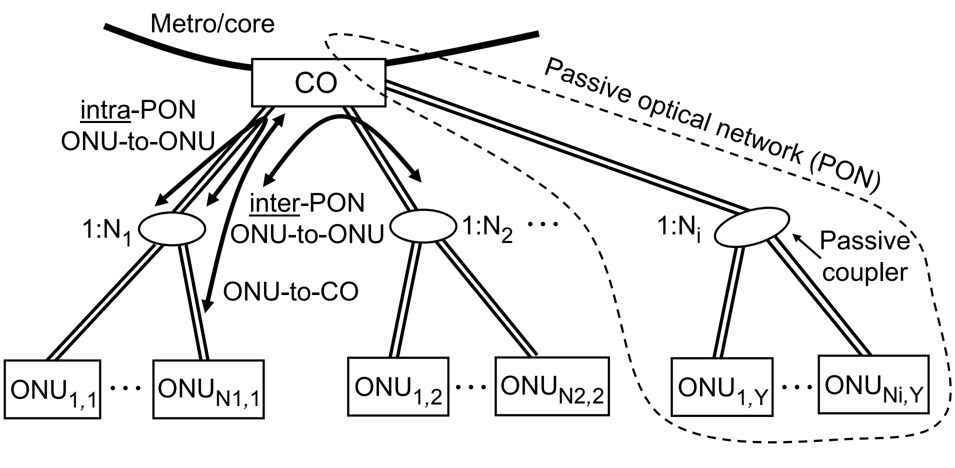

Figure 1 Access network tier based on PON including traffic streams

The ONUs in Figure 1 are uniquely identified by an optical address which is a combination of an optical code (OC) and a wavelength(-band). If the transmitted data is signed with the optical address, the optical carrier can be shared by all users in the network because within a set of OCs each code is designed to have high autocorrelation and low cross-correlation values, and optical code division multiple access (OCDMA) is enabled [2]. However, a soft capacity degradation is observed that is caused by multiple user interference (MUI) as a function of the number of active channels. Consequently, the average bit error rate (BER) experienced rises for increasing load, and error-correction techniques are required to support a high number of channels in such code-based network architecture.

1.2 Transparent ONU-to-ONU communication

Typically, all traffic is terminated at the CO, even in the case that ONUs in the same PON (sub-net) communicate with each other. Such ONU-to-ONU (O2O) communication normally requires a conversion to the electronic domain in order to perform the routing of the data. In prior work, transparency has been introduced in Figure 1 by transmitting O2O traffic on the optical address of the receiving node [1]. It is clear that contention has to be resolved at the merging nodes when two ONUs transmit data to a shared destination. In this work, these ONUs are considered to be attached to the same passive coupler.

Simple implementations to avoid collisions are the well-known ALOHA-based protocols. Here, a transmission scheduling protocol is proposed which requires upgrading the network with a reflective coupler and user monitoring functionalities at the ONU [1]. In that case, the ONU is able to derive accurate information on the network status and act accordingly. The analysis presented in this work is done at the packet level because the OC lasts the full duration each bit of a transmitted packet while similar work considers time-arranged pulses as OC and on-off keying [3]. In that case, the analysis has to be done on the bit-level which is more complicated.

2. Analysis

2.1 Notation and problem statement

Let us define the following system parameters:

- *M*: Maximum number of active optical codes allowed transmitting in the PON.
- *L*: The length of the fiber
- *B*: The bit rate of the link.

Essentially, at a given slotted time, no more than *M* ONUs can be simultaneously transmitting data. Hence, when a given ONU is willing to transmit, it must first check if the shared media is available or not, that is, if the number of active sources *u* at the PON is smaller than the maximum number permitted *M*. If so, it can transmit its data although for a high number of *u* the channel is usually not error-free due to MUI. In this work the noise-model in [1] is used to calculate the BER versus *u* for an incoherent spectral code system. In case *M* is reached, the ONU must wait until one of the ONUs finishes and releases the channel on the shared medium.

We define a time-slot *ts* as the (one-way) duration of a single packet. That is,

$$ts := \frac{\mu \times 8}{B} \qquad (1)$$

where μ is the mean value of the packet size.

In such a time-slotted scenario, each ONU is modeled as a two-state on/off system, whereby the "on" periods denote the time-slots at which a given ONU is transmitting its data, and the "off" periods refer to the time-slots where no data is sent by the ONU. Accordingly, let $X_{on} \geq 1$ denote the number of time slots that a given ONU occupies the channel.

Since the Geometric distribution is known to be memoryless, this shall be used to model the cases whereby incoming traffic at the ONUs is highly-multiplexed from different sources, hence traffic injected at the PON is therefore Poisson. However, this is not always the case and, since the ONUs might be residential end-users (not highly-multiplexed traffic), the traffic input process might show self-similar statistical properties. The heavy-tailed distribution proposed above models self-similarity for time-slotted systems, as shown in [4]. Therefore, in what follows, we shall assume the following two possible probability mass functions for X_{on} in case of a geometric and heavy-tailed distribution:

- Geometric distribution:

$$P(X_{\text{on}} = k) = p_{\text{on}}(1 - p_{\text{on}})^{k-1}, k = 1, 2, \ldots \quad (2)$$

  characterized by parameter p_{on}. This implies an average "on"-period duration of $EX_{on} = 1/p_{on}$.

- Heavy-tailed distribution:

$$P(X_{\text{on}} = k) = \frac{1}{k^{\alpha}} - \frac{1}{(k+1)^{\alpha}}, k = 1, 2, \ldots \quad (3)$$

characterized by tail-index α. This implies an average "on"-period duration of $EX_{on} = \zeta(x)$, where $\zeta(x)$ is the Riemann zeta-function.

The next subsection derives the blocking-time distribution for an OCDMA PON.

2.2 Blocking-time distribution

As previously stated, the OCDMA PON has many benefits, but its main disadvantage lays in the fact that no more than *M* ONUs can transmit simultaneously. If more ONUs than *M* want to make use of the shared media, the interference level at the feeder fiber results in erroneous transmission of all the ONUs and their data is lost. To avoid such shutdown of the system, it is key to study the blocking time distribution, that is, the number of time-slots whereby no user can send any data because the maximum number of ONUs are accessing the media simultaneously. Deriving the blocking probability distribution is used for analyze the feasibility of the transmission scheduling.

Let *D* denote the blocking-time distribution, that is, the distribution probability of the number of slots at which the system is blocked. To derive $P(D=k)$, $k=1,2,\ldots$ it is worth noticing that the system enters a blocked situation when the *M*-th connection arrives at the system. For example, consider the case of Figure 1 with *M*=4.

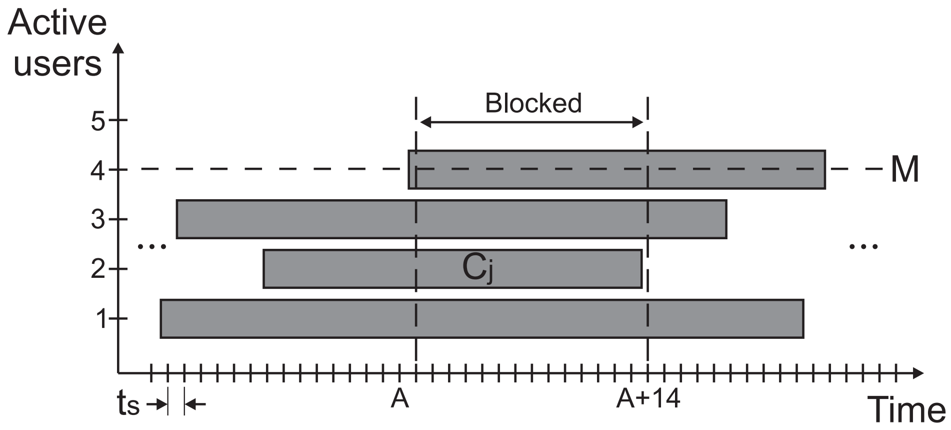

Figure 2: Example of blocked state with M=4.

The grey bars in Figure 2 indicate a burst of packets (X_{on}), and a negligible guard time between subsequent packets. As shown, access to the system is available (not blocked) during slotted time $k=A$ but at time $k=A+1$ a new connection arrives at the system. At this time, the PON enters the blocked state. The example shows that the system is blocked until $k=A+14$ since, at this moment, the ONU with code Cj finishes its transmission. Essentially, the system remains in the blocked duration until either any of the former $M-1$ ONUs finish, or the newly M-th arrival finishes, whichever occurs first.

As noted by Kleinrock [5], when the M-th connection arrives at the system, it sees the residual lives of the other $M-1$ connections. For a connection with random duration X, characterized by its probability distribution F_X and mean EX, its residual life is given by:

$$f_R(X) = \frac{1 - F_X(k)}{EX}, \quad k = 1, 2, \ldots \quad (4)$$

Hence, the blocking distribution, that is, the number of time slots that the system remains in the blocked period, is given by the minimum of the $M-1$ residual lives of the already transmitting ONUs and the duration of the M-th arrival. This is:

$$\begin{aligned} S_D(k) &= P(D > k) = P(\min\{R_1, \ldots, R_{M-1}, X\} > k) = S_{R_1}(k) \times \ldots \times S_{R_{M-1}}(k) \times S_X(k) \\ &= (S_{R_1}(k))^{M-1} S_X(k), \quad k = 0, 1, \ldots \end{aligned} \quad (5)$$

where $S_X(k)$ refers to the survival or complementary cumulative distribution function (CCDF) of random variable X, that is, $S_X(k)=1-F_X(k)$. Similarly, $S_{R_i}(k)$ refers to the CCDF of the residual life of connection i. The following derives the exact equations for S_D assuming both geometrically (memoryless) and heavy-tailed distributed “on” periods.

2.3 Geometric “on” periods

It is well-known that the residual life for a geometric probability distribution characterized with parameter p is geometrically distributed with parameter p [6]. Thus, the blocking CCDF duration, when the burst duration is geometrically distributed is given by:

$$S_D(k) = \left((1-p)^k\right)^{M-1} (1-p)^k = (1-p_{eq})^k, k = 1, 2, \ldots \quad (6)$$

where $p_{eq}=1-(1-p)^M$. Hence, the blocking probability distribution is also geometric with parameter p_{eq}, and average blocking duration $ED=1/p_{eq}$.

2.4 Heavy-tailed “on” periods

For heavy-tailed distributions with parameter α, the blocking distribution function is rather different. First of all, let us analyze the residual life observed by a random arrival:

$$f_R(k) = \frac{1-F_X(k)}{EX} = \frac{1}{\zeta(\alpha)}(1-(1-\frac{1}{(k+1)^\alpha})) = \frac{1}{\zeta(\alpha)}\frac{1}{(k+1)^\alpha}, \quad k = 0,1,\dots \quad (7)$$

Thus:

$$P(R = k \mid k \geq 1) = \frac{1}{\zeta(\alpha)-1}\frac{1}{(k+1)^\alpha}, \quad k = 1,2,\dots \quad (8)$$

whose CCDF is given by:

$$S_R(k) = \sum_{i=k+1}^{\infty} \frac{1}{\zeta(\alpha)-1}\frac{1}{(i+1)^\alpha}, \quad k = 1,2,\dots \quad (9)$$

Thus, the blocking CCDF duration, when the burst duration is heavy-tailed distributed is given by:

$$S_D(k) = \left(\frac{1}{\zeta(\alpha)-1}\sum_{i=k+1}^{\infty}\frac{1}{(i+1)^\alpha}\right)^{M-1}\frac{1}{(k+1)^\alpha}, k = 1,2,\dots \quad (10)$$

The average blocked duration is therefore given by the following sum:

$$ED = \sum_{k=1}^{\infty} S_D(k) = \frac{1}{(\zeta(\alpha)-1)^{M-1}}\sum_{k=1}^{\infty}\left(\sum_{i=k+1}^{\infty}\frac{1}{(i+1)^\alpha}\right)^{M-1} \times \frac{1}{(k+1)^\alpha} \quad (11)$$

with no closed form.

2.5 Coherence of state: Transmission delay vs. blocking duration

Let us assume that the propagation delay from one ONU to the passive coupler it is τ seconds, as shown in Figure 3. Therefore, the information sent by one ONU, it is sensed 2τ seconds in the future by the remaining ONUs as shown in Figure 3.

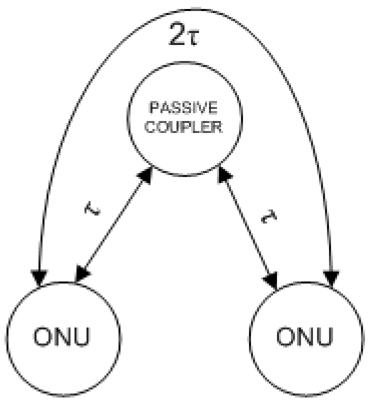

Figure 3: Transmission delay in a PON.

In order to do transmission scheduling, coherence is required between the sensed state at the ONU and the real or future state at the passive coupler. In other words, the blocking-duration must be greater than the transmission delay otherwise transmission scheduling does not make sense.

In order to evaluate the feasibility of transmission scheduling we consider $P(D>2\tau_{ts})$, where:

$$\tau_{ts} := \left\lceil \frac{\tau}{ts} \right\rceil = \left\lceil \frac{\tau B}{\mu \times 8} \right\rceil \quad (12)$$

is the transmission delay (only one way) expressed in time-slots.

We are interested in finding the optimum values for L, B, EX_{on} and M such that $P(D>2\tau_{ts})\geq 1-\varepsilon$. For instance, it is interesting to know, the maximum bit rate, B, such that $P(D>2\tau_{ts})\geq 0.8$ i.e. there is coherence of state 80% of the blocking time.

3. Numerical experiments

This section provides a set of numerical examples to study the feasibility of the transmission scheduling with respect to several network dimensioning parameters. In the time-slot computation (eq. 1), μ equals to 449.14 bytes which is determined to be the average empirical packet size [6].

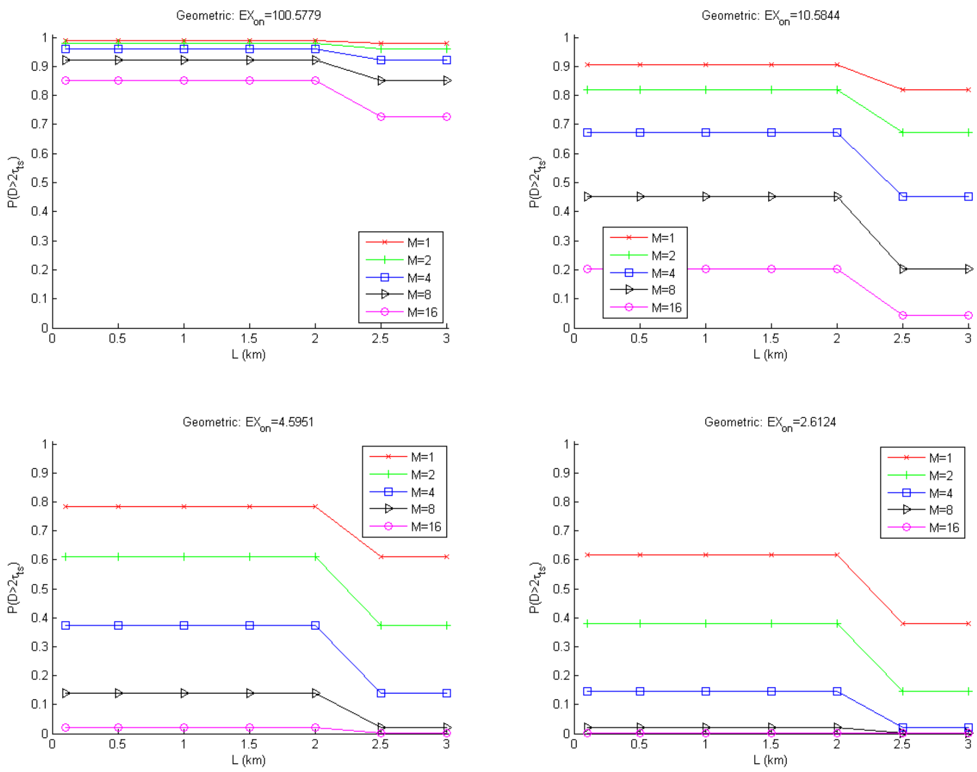

Figure 4: Length fiber vs. Coherence of state (Geometric distribution) for B=155 Mbps

Figure 4 shows the probability $P(D>2\tau_{ts})$ as a function of L (in km) for different values of M and EX_{on}, where X_{on} is geometrically distributed. The values considered are: L in (0, 3) km, M={2, 4, 8, 16} users, EX_{on}={100.58, 10.58, 4.59, 2.62} packets, which correspond to the expected values for the Pareto distribution with parameter α ={1.01, 1.1, 1.25, 1.5}, and B=155 Mbps (STM-1 bit rate, motivated in [1]).

As shown in Figure 4, $P(D>2\tau_{ts})$ does not depend on L for values up to 2 km, because τ is smaller than one time-slot, i.e. the packet duration is longer than the transmission delay. Therefore, the fiber length is not an important parameter (in terms of coherence of state) if the distance between the ONUs and the optical coupler is not greater than 2 km at the STM-1 bit rate. Furthermore, we can see that $P(D>2\tau_{ts})$ decreases as M increases since ED is smaller. Intuitively, blocking lasts less time when the number of active users is higher and therefore transmission scheduling has an upper bound in terms of M. Finally, Figure 4 shows that $P(D>2\tau_{ts})$ increases with an increasing EX_{on} since the value of ED increases, that is the blocking lasts more time when the bursts are longer. It can be concluded that long bursts should be used instead of short bursts of packets in order to preserve the coherence of state.

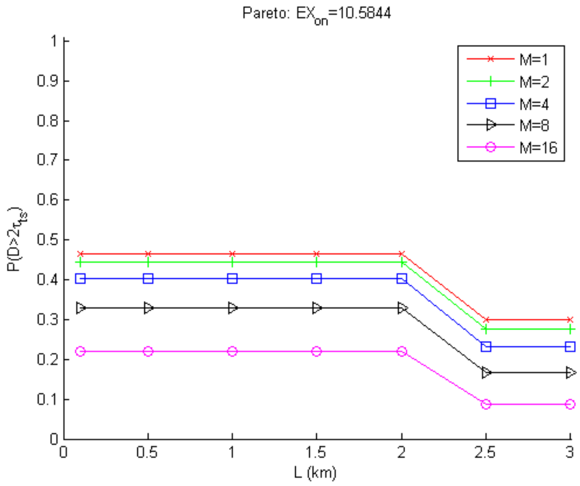

Figure 5: Fiber length vs. Coherence of state (Pareto distribution)

In Figure 5 it can be seen the results for the heavy-tailed distribution. The figure shows that the system behavior is similar to the geometric case. However, the values of $P(D>2\tau_{ts})$ are smaller because the variance of X_{on} is infinite in the Pareto case. Indeed, the obtained values for $P(D>2\tau_{ts})$ are too small to do transmission scheduling (smaller than 0.5). Therefore, if the traffic is not highly multiplexed, it is less interesting to deploy the transmission scheduling medium access mechanism.

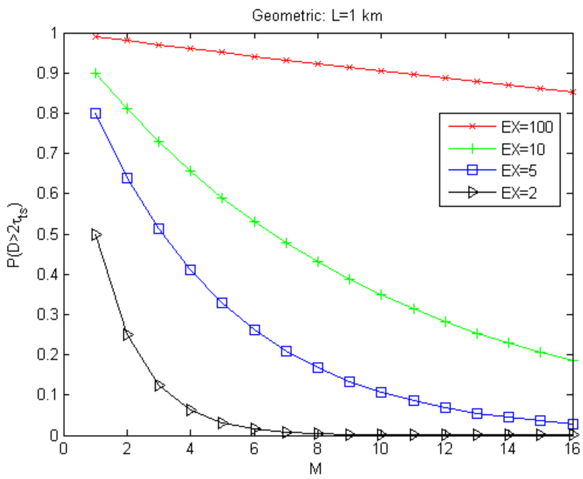

Figure 6: Maximum number of ONUs vs. Coherence of state

Figure 6 shows the probability $P(D>2\tau_{ts})$ as a function of the maximum number of active users in the PON, for different values of EX_{on}={100,10,5,2}, L=1km and X_{on} geometrically distributed. For instance, if we want to assure $P(D>2\tau_{ts})\geq 0.8$, the maximum number of active users must be less or equal than 2 if EX_{on} =10 packets whereas if EX_{on} =100 packets, then $P(D>2\tau_{ts})\geq 0.8$ for $M\leq 16$. Again, this indicates the need for long bursts in the network considered in this paper.

Figure 7 (left) shows the probability $P(D>2\tau_{ts})$ as a function of the mean value of the burst length, EX_{on}, for different values of M={2,4,8,16}, L=1km, B=155 Mbps and X_{on} geometrically distributed. Similarly to the results in Figure 4, $P(D>2\tau_{ts})$ increases as the burst length increases. For instance, If we want to assure $P(D>2\tau_{ts})\geq 0.8$, the expected value of the burst length has to be greater than 50 if *M*=8, whereas if *M*=2, the mean value of the burst length only has to be greater than 10.

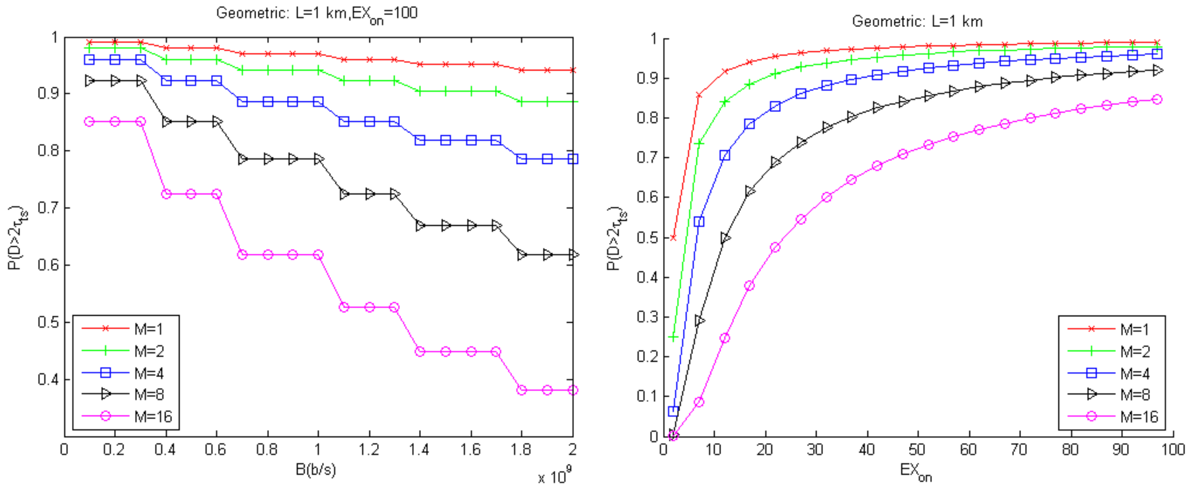

Figure 7: (left) Burst length vs. Coherence of state **(right)** Bit rate vs. Coherence of state

Finally, Figure 7 (right) shows the probability $P(D>2\tau_{ts})$ as a function of the bit rate of the channel, *B*, for different values of *M*={2,4,8,16}, *L*=1km, EX_{on}=100 and X_{on} geometrically distributed. We can see that $P(D>2\tau_{ts})$ decreases as *B* increases. For instance, If we want to assure $P(D>2\tau_{ts})\geq 0.8$, the bit rate must be less than 800 Mbps if *M*=8, whereas if *M*=2, the bit rate can reach up to 2Gbps.

4. Conclusions

This work provides a set of guidelines for designing PONs, assuming a set of topology constraints (fiber length) observed traffic patterns (EX_{on}) and maximum number of allowed simultaneous users. All these parameters determine the maximum achievable bit rate such that coherence of state holds, thus permitting transmission scheduling.

Acknowledgements

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